41514 – Dynamics of Machinery
– Theory, Experiment, Phenomenology and Industrial Applications –

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1. Course Structure
2. Objectives
3. Theoretical and Experimental Example
4. Industrial Applications
5. Mathematical Modeling & Steps
6. Examples
### Course Structure

#### Grade

\[ \text{Grade} = \left( \frac{1}{3} \right) \times \text{Project I} + \left( \frac{2}{3} \right) \times \text{Project II} \]

- **Project I** – Modeling & Experimental Validation via Experimental Modal Analysis
- **Project II** – FEM Modeling, Experimental Validation & Prediction of Rotor Stability
It is important to highlight that the “selected pages of some reference books and manuscripts” are very useful, if you attend the classes. The material is not ready yet to be used as a textbook and the structure of the course simulates the routine of a consulting engineer, who has to search for information in several sources in order to deal with the project challenges. In real life such information sources are frequently fragmented in several books and articles, and to facilitate and speed up the students’ learning process the sources are collected in the last column of the website. Remember though, that the discipline Dynamics of Machinery demands a strong integration of several disciplines, as dynamics, mechanical vibration, strength of materials, fluid mechanics, mathematics, numerical methods among others. There will be two educational projects and they were carefully formulated with the goal of integrating all these disciplines and strengthening the students’ background. While writing the final project reports all “apparently fragmented information” will be nicely linked together and the student will be able to understand Dynamics of Machinery in a holistic way, as it is in the reality.
2. Objectives – Multidisciplinary Approach

Physical System

- **Mass Elements M**
  - Particle
  - Rigid Body
  - Distributed

- **Spring Elements K**
  - Elasticity Theory & Material
  - Magnetism
  - Fluid Mechanics

- **Damping Elements D**
  - Fluid Mechanics
  - Contact Mechanics (friction)

Assumptions (simplifications)

Mathematical Model

- **Newton, Euler, D’Alembert, Lagrange, Hamilton, Jourdain**
  - (principles & axioms)

Mechanical Model

\[ \ddot{x}(t) = f(\dot{x}(t), x(t)) \]

**Static Equilibrium Position** (Linearization)

\[
M \ddot{x}(t) + D \dot{x}(t) + K x(t) = f(t) \quad \text{(structure)}
\]

\[
M \ddot{x}(t) + (D + G(\Omega)) \dot{x}(t) + K x(t) = f(t) \quad \text{(machine)}
\]

**Solution:**

\[
x(t) = \sum C_i u_i e^{\lambda_i t} + x_p(t)
\]

\[
\lambda(\Omega) = [\lambda_1 \lambda_2 \lambda_3 \ldots] \quad \text{(eigenvalues)}
\]

\[
U(\Omega) = [u_1 u_2 u_3 \ldots] \quad \text{(eigenvectors)}
\]
3. Theoretical and Experimental Example

**Physical System**

**Mechanical Model**

**Mathematical Model**

Eigenvalues:

$$\lambda(\Omega) = [\lambda_1 \quad \lambda_2 \quad \lambda_3 \ldots]$$

Eigenvectors:

$$U(\Omega) = [u_1 \quad u_2 \quad u_3 \ldots]$$

Application Examples:

- seals (rotor-stator contact)
- positioning of dampers
- gearboxes
4. Industrial Examples (Application)

**Tradicional Compressor**
(supported by fluid film bearings)

**Integrated Motorized Compressor**
(supported by magnetic bearings)

\[ M\ddot{x}(t) + (D + G(\Omega))\dot{x}(t) + Kx(t) = f(t) \]
5. Mathematical Modeling & Steps

Assumptions (simplifications)

Physical System

- *Mass Elements M*
  - Particle
  - Rigid Body
  - Distributed

- *Spring Elements K*
  - Elasticity Theory & Material
  - Magnetism
  - Fluid Mechanics

- *Damping Elements D*
  - Fluid Mechanics
  - Contact Mechanics (friction)

Mathematical Model

Newton, Euler, D’Alembert, Lagrange, Hamilton, Jourdain (principles & axioms)

\[ \ddot{x}(t) = f(\dot{x}(t), x(t)) \]

**Static Equilibrium Position** (Linearization)

\[
M \ddot{x}(t) + D \dot{x}(t) + K x(t) = f(t) \quad \text{(structure)}
\]

\[
M \ddot{x}(t) + (D + G(\Omega)) \dot{x}(t) + K x(t) = f(t) \quad \text{(machine)}
\]

Solution:

\[
x(t) = \sum C_i \mathbf{u}_i e^{\lambda_i t} + \mathbf{x}_p(t)
\]

\[
\lambda(\Omega) = \left[ \lambda_1 \lambda_2 \lambda_3 \ldots \right] \quad \text{(eigenvalues)}
\]

\[
\mathbf{U}(\Omega) = \left[ \mathbf{u}_1 \mathbf{u}_2 \mathbf{u}_3 \ldots \right] \quad \text{(eigenvectors)}
\]
5. Mathematical Modeling & Steps

- **Kinematics**
  1. Reference Frames (systems of coordinates)
  2. Transformation Matrices
  3. Position Vectors
  4. Velocity Vectors (linear and angular)
  5. Acceleration Vectors (linear and angular)

- **Dynamics**
  6. Mass Properties (mass center, moments of inertia)
  7. Force and Moment Vectors
  8. Dynamic Equilibrium: \( \ddot{x}(t) = f(\dot{x}(t), x(t)) \) (Newton, Euler, Lagrange …)
  9. Equilibrium Poition (Linearization, Vibration Analysis): \( M\ddot{x}(t) + D\dot{x}(t) + Kx(t) = f(t) \)

* **Mass Elements M**
  Particle
  Rigid Body
  Distributed

* **Spring Elements K**
  Elasticity Theory & Material
  Magnetism
  Fluid Mechanics

* **Damping Elements D**
  Fluid Mechanics
  Contact Mechanics (friction)
6. Examples – Particle in 3D (Three Equations)
GOAL: **Two Dynamic Reaction Forces** & **One Equations of Motion**
(dimensioning & design)                                               (stability analysis and vibrations)                  (machine elements, strength of materials)                  (mechanical vibrations)

- Dynamic reaction forces:
  
  direction $X_3$ : $R = m(b\ddot{a}^2 \sin \beta + 2l\dot{\psi}(\dot{\alpha} + \dot{\beta}) \cos \psi)$
  
  direction $Z_3$ : $T = m\{g \cos \psi + [b\ddot{a}^2 \cos \beta + r(\ddot{\alpha} + \ddot{\beta})^2] \sin \psi + l[\dot{\psi}^2 + (\dot{\alpha} + \dot{\beta})^2 \sin^2 \psi]\}$

- Equation of motion for the particle $E$ (direction $Y_3$)

  direção $Y_3$ : $\ddot{\psi} + \frac{g}{l} \sin \psi - \frac{(b\ddot{a}^2 \cos \beta + r(\ddot{\alpha} + \ddot{\beta})^2)}{l} \cos \psi - (\dot{\alpha} + \dot{\beta})^2 \sin \psi \cos \psi = 0$
GOAL: Two Dynamic Reaction Forces & One Equations of Motion
(dimensioning & design) (stability analysis and vibrations)
(machine elements, strength of materials) (mechanical vibrations)

6. Examples – Particle in 3D (Three Equations)