Ilmar Ferreira Santos

1. Recapitulation

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3. Experimental Modal Analysis: Another Experimental Example

4. Understanding the Frequency Response Functions (FRF): Resonances, Anti-Resonances, Phase (Delay), Coherence

5. Understanding Mode Shapes & Anti-Resonances: Experimental Visualization of Modes Shapes

6. Extracting Modal Parameters: From Steady-State Response (FRF) From Transient Vibrations

7. Application of Computer Models
1. Recapitulation – Mathematical Modeling & Steps

Physical System

Assumptions (simplifications)

Mechanical Model

Newton, Euler, D'Alembert, Lagrange, Hamilton, Jourdain (principles & axioms)

Mathematical Model

* Mass Elements M
  Particle
  Rigid Body
  Distributed

* Spring Elements K
  Elasticity Theory & Material Magnetism
  Fluid Mechanics

* Damping Elements D
  Fluid Mechanics
  Contact Mechanics (friction)

• Kinematics
  1. Reference Frames (systems of coordinates)
  2. Transformation Matrices
  3. Position Vectors
  4. Velocity Vectors (linear and angular)
  5. Acceleration Vectors (linear and angular)

• Dynamics
  6. Mass Properties (mass center, moments of inertia)
  7. Force and Moment Vectors
  8. Dynamic Equilibrium: \[ \ddot{x}(t) = f(\dot{x}(t), x(t)) \]
  9. Equilibrium Position (Linearization, Vibration Analysis)
     \[ M\ddot{x}(t) + D\dot{x}(t) + Kx(t) = f(t) \]
1. Recapitulation – Experimental Validation of Models

Simulation Model
(Theory of Machinery Dynamics)

- Physical System
- Mechanical Model
- Mathematical Model
  - Assumptions (simplifications)
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* Mass Elements M
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  - Rigid Body
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* Damping Elements D
  - Fluid Mechanics
  - Contact Mechanics (friction)

\[ \ddot{x}(t) = f(x(t), \dot{x}(t)) \]

Static Equilibrium Position (Linearization):
\[ \ddot{x}(t) + D\dot{x}(t) + Kx(t) = f(t) \] (structure)
\[ \ddot{x}(t) + (D + G(t))\dot{x}(t) + Kx(t) = f(t) \] (machine)

Solution:
\[ x(t) = \sum C_i \phi_i e^{\lambda_i t} + x_0(t) \]

\[ \lambda(t) = [\lambda_1, \lambda_2, \lambda_3, \ldots] \] (eigenvalues)
\[ U(t) = [u_1, u_2, u_3, \ldots] \] (eigenvectors)

Undamped natural frequencies
Damped natural frequencies
Damping factors
Mode shapes
(Theoretical Modal Analysis)

Industrial Application of Computer Models: design, optimization, stability, …

Experimental Validation
(Experimental Modal Analysis)

Undamped natural frequencies
Damped natural frequencies
Damping factors
Mode shapes
(Experimental Modal Analysis)
The main goals of this project are:

- To build **mechanical** and **mathematical** models and computationally implement them for simulating the **dynamical behavior** of rotating machines, mechanisms and structures.

- To deal with **multibody systems** and **finite element method** to create the models and describe the dynamic behavior of **rigid and flexible** machine/structure components.

- To understand the **connections among different disciplines**, as dynamics, mechanical vibrations, strength of materials, experimental mechanics, signal processing, mathematics and numerical analysis.

- To obtain the coefficients of differential equations of motions which may be constant or depending on angular velocity of the machines. **In project 1 such coefficients will be kept constant after the linearization of the equations of motion, leading to mathematical models with constant eigenvalues and eigenvectors.** In project 2 such coefficients will be dependent on the angular velocity of the machines, and the eigenvalues and eigenvectors will vary as function of the system operation conditions.

- To deal theoretically as well as experimentally with damped systems, natural mode shapes, natural frequencies, damped natural frequencies and damping factors are calculated.

- To visualize and understand the physical meaning of natural mode shapes, natural frequencies, modal coordinates, modal mass, modal stiffness, modal damping among others when such parameters are not depending on the system operational conditions, for example, angular velocity of the machines.
2. Project Overview: Step by Step

- To understand the principles of experimental dynamic testing and the operational principles of sensor and actuators, among them accelerometers, displacement and force transducers and electromagnetic shakers.

- To understand the techniques of signal analysis and processing, which make possible the development of the experimental methodologies for validating mathematical models.

- To understand and use signal processing techniques to obtain auto and cross-correlation functions, power and cross-spectral density functions, frequency response functions and coherence function and, finally, demonstrate practical experience in extracting modal parameters from frequency response functions.

- To validate mathematical models based on Experimental Modal Analysis (EMA). Remember, if the measured frequencies and mode shapes agree with those predicted by the analytical mathematical model, the model is verified and can be useful for design proposes and vibration predictions with some confidence. Otherwise, the analytical models are useless.

- To account for the limitations in the models and methods used, and predict the possible consequences of making simplified assumptions.

- To write technical reports, with correct description of theoretical and experimental procedures, in a clear way, well structured language, using technical terms, giving physical interpretations and evaluations of analytical, numerical and experimental results.
3. Experimental Modal Analysis: Another Example

**Physical System**

**Mechanical Model**

**Mathematical Model**

- *Mass Elements M*
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  - Rigid Body
  - Distributed

- *Spring Elements K*
  - Elasticity Theory & Material
  - Magnetism
  - Fluid Mechanics

- *Damping Elements D*
  - Fluid Mechanics
  - Contact Mechanics (friction)

**Assumptions (simplifications)**

- Newton, Euler, D’Alembert, Lagrange, Hamilton, Jourdain (principles & axioms)

**Kinematics**
1. Reference Frames (systems of coordinates)
2. Transformation Matrices
3. Position Vectors
4. Velocity Vectors (linear and angular)
5. Acceleration Vectors (linear and angular)

**Dynamics**
6. Mass Properties (mass center, moments of inertia)
7. Force and Moment Vectors
8. Dynamic Equilibrium: \( \ddot{\mathbf{x}}(t) = \mathbf{f}(\dot{\mathbf{x}}(t), \mathbf{x}(t)) \) (Newton, Euler, Lagrange ...)
   \[ M\ddot{\mathbf{x}}(t) + D\dot{\mathbf{x}}(t) + K\mathbf{x}(t) = \mathbf{f}(t) \]

\[ \lambda(\Omega) = \begin{bmatrix} \lambda_1 & \lambda_2 & \lambda_3 & \ldots \end{bmatrix} \quad \text{(eigenvalues)} \]

\[ \mathbf{U}(\Omega) = \begin{bmatrix} \mathbf{u}_1 & \mathbf{u}_2 & \mathbf{u}_3 & \ldots \end{bmatrix} \quad \text{(eigenvectors)} \]
3. Experimental Modal Analysis: Another Example
4. Understanding FRF: Correlation, Power Spectrum Density, FRF, Coherence
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5. Understanding Mode Shapes & Anti-Resonances

\[ \lambda(\Omega) = [\lambda_1 \ \lambda_2 \ \lambda_3 \ \ldots] \] (eigenvalues)

\[ U(\Omega) = [u_1 \ u_2 \ u_3 \ \ldots] \] (eigenvectors)
6. Extracting Modal Parameters

Transient Vibrations

Steady-State Vibrations
6. Extracting Modal Parameters – Steady-State Responses (FRF)

From modal mass $m$, modal stiffness $k$ and modal damping $d$ to natural frequency $\omega$ & damping factor $\xi$

Remember that after experimentally calculating the coefficients $m_i$, $k_i$, and $d_i$ around the six resonance ranges, i.e. $i = 1, 2, 3, 4, 5, 6$, the natural frequencies can be obtained by $\omega_i = \sqrt{\frac{k_i}{m_i}}$ and $\xi_i = \frac{d_i}{2\sqrt{m_i k_i}}$. 
6. Extracting Modal Parameters – Transient Vibrations

a) Least Square Method

b) Half Power Points

\[ Q = \frac{1}{2\zeta} = \frac{\omega_2 - \omega_1}{\omega_2} \]

Log Dec. / Damping Ratio

\[ \xi = \frac{1}{2\pi N} \ln \left( \frac{y_0}{y_N} \right) \]

\[ \sqrt{1 + \left[ \frac{1}{2\pi N} \ln \left( \frac{y_0}{y_N} \right) \right]^2} \]

Half Power Points

\[ \frac{d^2 + 2\zeta \omega_n \dot{\chi}_n + \omega_n^2 \chi_n}{m} = 0 \]

characteristic polynomial

\[ \chi_{1,2} = \frac{-d \pm \sqrt{d^2 - 4mk}}{2m} \]

\[ d^2 - 4mk = 0 \quad \text{critical damping} \quad \chi_{1,2} = \frac{d}{2m} \]

\[ d^2 - 4mk > 0 \quad \text{overdamped} \quad \chi_{1,2} < 0 \]

\[ d^2 - 4mk < 0 \quad \text{underdamped} \quad \chi_{1,2} = -\lambda \pm i\omega_n \]

\[ \omega_n = \sqrt{\frac{4mk}{m}} \]

\[ \zeta = \frac{1}{2\pi N} \ln \left( \frac{y_0}{y_N} \right) \]

\[ \sqrt{1 + \left[ \frac{1}{2\pi N} \ln \left( \frac{y_0}{y_N} \right) \right]^2} \]

Critical damping: \( d = 2\sqrt{mk} \)

\[ \chi(t) = C_1 e^{\xi t} + C_2 e^{\frac{-d}{2m} t} \]

\[ \chi(t) = C_0, e^{-\xi t} \cos \left( \omega_n \sqrt{1 - \xi^2} t + \phi \right) \]
Why Have Different Methods

Time or frequency - which to use???

It really depends on which domain has the most data
7. Application of Computer Models

Matlab Rotines – dof2_frf.
dof2_modal_analysis.m

Application of Computer Models:
prediction of maximum deformations,
design verification,
re-dimensioning,
opimization …