41514 – Dynamics of Machinery
– Theory, Experiment, Phenomenology and Industrial Applications –

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2. Educational Application of Experimental Modal Analysis
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1. Industrial Application of Experimental Modal Analysis

Application of Modal Analysis to Aerospace Industry
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STANDARDS:
ASTM D4123
ASTM D3497
BS 598 DD
SHRP A003A
2. Application of Experimental Modal Analysis
Review of Some of the Definitions Used in Random Vibration Analysis

The autocorrelation function of the random signal \( x(t) \) is given by

\[
R_{xx}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_0^T x(t)x(t + \tau) \, dt
\]

(3.50)

The power spectral density (PSD) of a signal is the Fourier transform of the signal's autocorrelation:

\[
S_{xx}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R_{xx}(\tau)e^{-i\omega \tau} \, d\tau
\]

(3.51)

as indicated by equation (3.62) and reviewed in Window 7.3. Equation (7.7) relates the dynamics of the test structure contained in \( H(\omega) \) to measurable quantities (i.e., the PSDs). As pointed out at the end of Section 3.7, the common approach to measuring the frequency response function is to average several matched sets of input force time histories and output response time histories. These averages are used to produce correlation functions that are transformed to yield the corresponding PSDs. Equation (7.7) is then used to calculate the magnitude of the frequency response function \(|H(\omega)|\). The experimental vibration data are then taken from the plot of \(|H(\omega)|\) as indicated in Figure 3.15, or by means to be discussed in Section 7.4.

The frequency response function can also be related to the cross-correlation between the two signals \( x(t) \) and \( f(t) \). The cross-correlation function, denoted \( R_{xf}(\tau) \), for the two signals \( x(t) \) and \( f(t) \), is defined by

\[
R_{xf}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_0^T x(t)f(t + \tau) \, dt
\]

(7.8)

Here \( x(t) \) is considered to be the response of the structure to the driving force \( f(t) \). Similarly, the cross-spectral density is defined as the Fourier transform of the cross-correlation:

\[
S_{xf}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R_{xf}(\tau)e^{-i\omega \tau} \, d\tau
\]

(7.9)
These correlation and density functions also allow calculation of the transfer functions of test structures. The frequency response function, $H(j\omega)$, can be shown (see, e.g., Ewins, 1984) to be related to the spectral density functions by the two equations

$$S_{fx}(\omega) = H(j\omega)S_{ff}(\omega)$$ (7.10)

and

$$S_{xx}(\omega) = H(j\omega)S_{xf}(\omega)$$ (7.11)

These hold if the structure is excited by a random input $f(t)$ resulting in the response $x(t)$. Note that the cross-correlation functions include information about the phase and magnitude of the structure’s transfer function and not just the magnitude, as in the case of the correlation function of equation (7.7) repeated on the bottom right corner of Window 7.3.

The spectrum analyzer calculates (or estimates) the various spectral density functions from the transducer outputs. Then, using equation (7.10) or (7.11), the

$$H_1(\omega) = \frac{S_{fx}(\omega)}{S_{ff}(\omega)}$$

better estimator (anti-resonance)

$$H_2(\omega) = \frac{S_{xx}(\omega)}{S_{xf}(\omega)}$$

better estimator (resonance)
3. Frequency Response Functions – H1, H2 & Coherence

Figure 7.6 Sample plot of a coherence function.

The coherence function, denoted by \( \gamma^2 \), is defined to be the ratio of the two values of \( H(j\omega) \) calculated from equations (7.10) and (7.11). In particular, the coherence function is defined to be

\[
\gamma^2 = \frac{|S_{xf}(\omega)|^2}{S_{xx}(\omega)S_{ff}(\omega)}
\]  

which always lies between 0 and 1. In fact, if the measurements are consistent, \( H(j\omega) \) should be the same value, independent of how it is calculated, and the coherence should be 1 (\( \gamma^2 = 1 \)). The coherence is a measurement of the noise in the signal. If it is zero, the measurement is of a pure noise; if the value of the coherence is 1, the signals \( x \) and \( f \) are not contaminated with noise. In practice, coherence versus frequency is plotted versus frequency (see Figure 7.6) and is taken as an indication of how accurate the measurement process is over a given range of frequencies. Generally, the values of \( \gamma^2 = 1 \) should occur at values of \( \omega \) far from the structure’s resonant frequencies. Near resonance the signals are large and magnify the noise. In practice, data with a coherence of less than 0.75 are not used and indicate that the test should be done over.